Kingsway Christian College

Maths Department

Course: Math Methods unit 3

Assessment Task: Test 2

Student Name: Solution Key.

Date: 23rd & 24th March 2017

Assessment Score: ____________ / 50

Year Score: ____________

Comments: ____________

Teacher signature: ____________

Parent/ Guardian signature: ____________

Comments: ____________

Question 2 **(6 marks)**

Determine the maximum and minimum value for $f(x)$ and the value of x at which they occur, for the function $f(x) = 3x^4 - 16x^3 + 18x^2$ over the domain $-1 \leq x \leq 2$.

$f(-1) = 3+16+18$
$= 37$

$f(2) = 48-128+72$
$= -8$

6

$\therefore f'(x) = 12x^3 - 48x^2 + 36x$
$= 12x(x^2 - 4x + 3)$
$= 12x(x-3)(x-1)$ ✓

for max/min:

$f'(x) = 0$ ✓ $\Rightarrow x = 0$ or $x = 1$ or $x = 3$

$f(0) = 0$ $f(1) = 5$ $\downarrow$ N.A.

$\downarrow$ $\downarrow$

$f''(0) = 0$ ✓ $f''(0) < 0$ ✓
p.o.i local max.

$\therefore$ Min : $(2; -8)$ ✓

Max : $(-1; 37)$ ✓

Question 3 **(7 marks)**

Determine the coordinates of all intercepts, stationary points and points of inflection of the function $y = x\,e^{3x}$.

Justify the nature of the stationary points found using a standard test.

$$\frac{dy}{dx} = e^{3x} + x \cdot e^{3x} \cdot 3$$
$$= e^{3x}(1+3x).$$

for min/max: $\frac{dy}{dx} = 0$

$\therefore e^{3x}(1+3x) = 0$

$\therefore x = -\frac{1}{3}$

$y = -\frac{1}{3}e^{-1}$

$= -\frac{1}{3e}$

$\therefore \left(-\frac{1}{3}; -\frac{1}{3e}\right)$.

and $\left.\frac{d^2y}{dx^2}\right|_{x=-\frac{1}{3}} = 3e^{-1}(2-1)$

$= \frac{3}{e}$

> 0

$\therefore$ min

$$\frac{d^2y}{dx^2} = 3e^{x}(1+3x) + e^{3x} \cdot 3$$
$$= 3e^{3x}(2+3x)$$

for P.O.I $\frac{d^2y}{dx^2} = 0$

$\Rightarrow x = -\frac{2}{3}$.

$\therefore y = \frac{-2}{3}e^{-2}$

$= -\frac{2}{3e^2}$

$\therefore \left(-\frac{2}{3}; -\frac{2}{3e^2}\right)$

and

x intercept: $(0;0)$.

7

Question 4 **(3 marks)**

Determine the equation of the normal to the curve $y = x(3-x)^2$ at (2,2).

$$\frac{dy}{dx} = 1(3-x)^2 + x(-2)(3-x).$$

$$\therefore \frac{dy}{dx} = (3-x)^2 - 2x(3-x)$$

3

$$\left.\frac{dy}{dx}\right|_{x=2} = (3-2)^2 - 2(2)(3-2)$$
$$= 1-4$$
$$= -3.$$

$\therefore y = -3x + c$

tangent $m = -3$

$\therefore$ normal $m = \frac{1}{3}$.

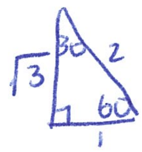
$y = \frac{1}{3}x + c$

$2 = \frac{1}{3}(2) + c \quad \therefore c = \frac{4}{3}$

$\therefore y = \frac{1}{3}x + \frac{4}{3}.$

Question 5 **(5 marks)**

Find the equation of the tangent to the curve $y = 2x + cos2x$ at the point $(\frac{\pi}{3}; \frac{2\pi}{3} - \frac{1}{2})$



$$\therefore \frac{dy}{dx} = 2 - 2\sin 2x.$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{3}} = 2 - 2\sin\frac{2\pi}{3}.$$
$$= 2 - 2\frac{\sqrt{3}}{2}$$
$$= 2 - \sqrt{3} = m.$$

$\therefore y = mx + c$

(Q5)

$$\therefore \quad \frac{2\pi}{3} - \frac{1}{2} = (2-\sqrt{3})\frac{\pi}{3} + C$$

5

$$\cancel{\frac{2\pi}{3}} - \frac{1}{2} - \cancel{\frac{2\pi}{3}} + \frac{\sqrt{3}\pi}{3} = C$$

$$\therefore C = \frac{\sqrt{3}\pi}{3} - \frac{1}{2}.$$

$$\therefore y = (2-\sqrt{3})x + \frac{\sqrt{3}}{3}\pi - \frac{1}{2}.$$

Math Methods Unit 3 Test 2 2017 Differentiation

Name Sol key.

Resource Assumed **Time: 25 minutes** **Marks: / 23**

CAS calculator and a formula sheet are allowed for this section

Question 6 **(5 marks)**

A cylindrical can is to be made to hold 1 000 cm^3 of oil. Find the dimensions that will minimise the amount of the metal to make the can. Assume the can is made with a lid.

$V = \pi r^2 h$ $\qquad SA = 2\pi r^2 + 2\pi r h.$

$\therefore 1000 = \pi r^2 h$

$\therefore h = \dfrac{1000}{\pi r^2}$ $\qquad \therefore SA = 2\pi r^2 + 2\pi r\left(\dfrac{1000}{\pi r^2}\right)$ ✓

$\therefore SA = 2\pi r^2 + \dfrac{2000}{r}.$

for min $SA' = 0.$

5

$SA' = 4\pi r - \dfrac{2000}{r^2}$ ✓

for min : $4\pi r - \dfrac{2000}{r^2} = 0$

$\therefore r = \sqrt[3]{\dfrac{500}{\pi}}$ ✓

$= 5.42$ cm. ✓ (calculator)

and $h = \dfrac{1000}{\pi r^2} = \dfrac{1000}{\pi (5.42)^2}$ ✓

$= 10.84$ cm.

Question 7 **(9 marks)**

The cost in dollars of producing x items is given by: $C(x) = (3000 + 5x)$.

The revenue per item sold is given by $\$(40 - 0.02x)$.

(a) State the revenue function $R(x)$ for the number of x items sold. (1 mark)

$R(x) = x(40 - 0{,}02x)$

1

(b) Give an expression for the profit function $P(x)$. (1 mark)

1

$P(x) = R(x) - C(x)$

$= x(40 - 0{,}02x) - (3000 + 5x)$

$\therefore P(x) = -0{,}02x^2 + 35x - 3000$

(c) Determine how many items are needed to make a maximum profit and state the maximum profit. (3 marks)

for max $P'(x) = 0$.

3

$\therefore P'(x) = -0{,}04x + 35 = 0$

$\therefore x = 875$.

$\therefore$ max Profit $= -0{,}02(875)^2 + 35(875) - 3000$

$= \$12\,312{,}50$

(d) Explain clearly if a loss occurred and when it occurred. (2 marks)

a loss occured when $P(x) < 0$.

2

(solve $(-0{,}02x^2 + 35x - 3000 < 0, x)$).

$\therefore \; 0 \leq x \leq 90$ or $x \geq 1660$.

(e) Determine the marginal profit of the 250th item sold. (2 marks)

$\therefore$ 249 are sold allready.

$\therefore P'(x) = -0{,}04x + 35$

$P'(249) = \$25{,}04$

2

Question 8 **(4 marks)**

Use derivatives to find the approximate change in the radius of a spherical balloon corresponding to a change in its volume from 200 cm^3 to 195 cm^3. Answer to 4 decimal places.

$V = \frac{4}{3}\pi r^3$ $\qquad \delta V = -5.$

4

$\frac{dV}{dr} = 4\pi r^2$

for incremental changes: $\frac{\delta V}{\delta r} \approx \frac{dV}{dr}$

$V = \frac{4}{3}\pi r^3$

$200 = \frac{4}{3}\pi r^3$

$r^3 = \frac{150}{\pi}$

$r^2 = \left(\frac{150}{\pi}\right)^{\frac{2}{3}}$

$\frac{\delta V}{\delta r} \approx 4\pi r^2 \quad \Big| r^2 = \left(\frac{150}{\pi}\right)^{\frac{2}{3}}.$

$\frac{\delta r}{\delta V} \approx \frac{1}{4\pi r^2}.$

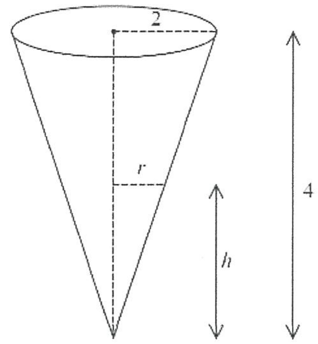
$\delta r \approx \frac{1}{4\pi r^2} \times \delta V$

$\approx \frac{1}{4\pi\left(\frac{150}{\pi}\right)^{\frac{2}{3}}} \times -5.$

$\delta r \approx \underline{-0.0302}$ cm.

Question 9 **(5 marks)**

A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m.



(a) Proof that the volume of the tank is given by the following formula:
$V(h) = \frac{1}{12}\pi h^3$ (1 mark)

$V = \frac{1}{3}\pi r^2 h$. and $r = \frac{h}{2}$.

$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \times h$

$V = \frac{1}{12}\pi h^3$

1

(b) If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.
Answer to the nearest cm/min. (4 marks)

$\frac{dV}{dt} = 2 m^3/min$

$\frac{dh}{dt} = ?$ $h = 3m$.

$\therefore \frac{dV}{dh} = \frac{3}{12}\pi h^2$

$= \frac{1}{4}\pi h^2 = \frac{\pi h^2}{4}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

4

$= \frac{4}{\pi h^2} \times 2$

$= \frac{8}{\pi h^2}$

$\frac{dh}{dt} = \frac{8}{\pi(3)^2} = 0{,}28 m/min.$ or $28 cm/min$